1. Funktion 3. Grades, 3 Punkte + 1 Ableitungswert von $f'(x)$

$I. f(2)=5$; $II. f(0)=7$; $III. f(3)=0$; $IV. f'(2)=0$

Ziel $\begin{pmatrix} \square & \square & \square & | & \square \\ D & \square & \cancel{②} & | & \square \\ \square & \cancel{③} & ① & | & \square \end{pmatrix}$

Ansatz: $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$I.$ $a \cdot 2^3 + b \cdot 2^2 + c \cdot 2 + d = 5$

$8a + 4b + 2c + d = 5$

$II.$ $a \cdot 0 + b \cdot 0 + c \cdot 0 + d = 7$ $\Rightarrow d = 7$ schon gelöst, kommt nicht in die Matrix

$III.$ $a \cdot 3^3 + b \cdot 3^2 + c \cdot 3 + 7 = 0 \,|-7$

$27a + 9b + 3c = -7$

$IV.$ $3a \cdot 2^2 + 2b \cdot 2 + c = 0$

$3a \cdot 4 + 2b \cdot 2 + c = 0$

$12a + 4b + c = 0$

I $\quad 8a + 4b + 2c + d = 5$

$\quad 8a + 4b + 2c + 7 = 5 \,|-7$

$I. \quad 8a + 4b + 2c = -2$

$III. \quad 27a + 9b + 3c = -7$

$IV. \quad 12a + 4b + c = 0$

$\begin{pmatrix} 8 & 4 & 2 & | & -2 \\ 27 & 9 & 3 & | & -7 \\ 12 & 4 & ① & | & 0 \end{pmatrix}$

3. Beispiel: Funktion 3. Grades, 3 Punkte + 1 Ableitungswert von $f'(x)$

I. $f(2)=5$; II. $f(0)=7$; III. $f(3)=0$; IV. $f'(2)=0$

Ziel $\begin{pmatrix} \square & \square & \square & | & \square \\ \square & \square & \cancel{\square} & | & \square \\ \square & \cancel{\square} & \cancel{\square} & | & \square \end{pmatrix}$

Ansatz: $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

I. $a \cdot 2^3 + b \cdot 2^2 + c \cdot 2 + d = 5$
$8a + 4b + 2c + d = 5$

II. $a \cdot 0 + b \cdot 0 + c \cdot 0 + d = 7 \Rightarrow d = 7$ schon gelöst, kommt nicht in die Matrix

III. $a \cdot 3^3 + b \cdot 3^2 + c \cdot 3 + 7 = 0 \,|\, -7$
$27a + 9b + 3c = -7$

IV. $3a \cdot 2^2 + 2b \cdot 2 + c = 0$
$3a \cdot 4 + 2b \cdot 2 + c = 0$
$12a + 4b + c = 0$

I. $8a + 4b + 2c + d = 5$
$8a + 4b + 2c + 7 = 5 \,|\, -7$

I. $8a + 4b + 2c = -2$
III. $27a + 9b + 3c = -7$
IV. $12a + 4b + c = 0$

$\begin{pmatrix} 8 & 4 & 2 & | & -2 \\ 27 & 9 & 3 & | & -7 \\ 12 & 4 & \textcircled{1}_1 & | & 0 \end{pmatrix} \cdot 3$

$\begin{pmatrix} 8 & 4 & 2 & | & -2 \\ 27 & 9 & \textcircled{3}_2 & | & -7 \\ 9 & 3 & 0 & | & 7 \end{pmatrix} \begin{matrix} \cdot 3 \\ \cdot 2 \\ \end{matrix}$

$\begin{pmatrix} 24 & 12 & 6 & | & -6 \\ -30 & -6 & 0 & | & 8 \\ 9 & \textcircled{3}_3 & 0 & | & 7 \end{pmatrix} \cdot 2$

$\begin{pmatrix} 8 & 4 & 2 & | & -2 \\ 27 & 9 & 3 & | & -7 \\ 36 & 12 & 3 & | & 0 \end{pmatrix}$

$\begin{pmatrix} 24 & 12 & 6 & | & -6 \\ 54 & 18 & 6 & | & -14 \\ 9 & 3 & 0 & | & 7 \end{pmatrix}$

$\begin{pmatrix} 24 & 12 & 6 & | & -6 \\ -30 & -6 & 0 & | & 8 \\ 18 & 6 & 0 & | & 14 \end{pmatrix}$

$$
\begin{array}{ccc}
a & b & c \\
\end{array}
$$

$$
\left(\begin{array}{ccc|c}
24 & 12 & 6 & -6 \\
-30 & -6 & 0 & 8 \\
-12 & 0 & 0 & 22
\end{array}\right)
$$

$-12a = 22 \mid : -12$

$a = \dfrac{22}{-12} = -\dfrac{11}{6}$

$-30 \cdot \dfrac{-11}{6} - 6b = 8$

$55 - 6b = 8 \mid -55$

$-6b = -47 \mid : (-6)$

$\cdot \, b = \dfrac{+47}{6}$

$24 \cdot \left(-\dfrac{11}{6}\right) + 12 \cdot \dfrac{47}{6} + 6c = -6$

$-44 + 94 + 6c = -6$

$50 + 6c = -6 \mid -50$

$6c = -56 \mid : 6$

$c = -\dfrac{28}{3}$

weitere Beispiele

a) $f(-4) = 0{,}6$; $f(2) = 0{,}6$; $f(1) = -0{,}4$ hoch 2

b) $f(0) = -1$; $f(2) = 15$; $f(-1) = -15$; $f(3) = 41$ hoch 3

c) $f(3) = 11$; $f(0) = 2$; $f'(1) = 1$ hoch 2 + f'

d) $f(0) = -3$; $f(2) = -7{,}8$; $f(3) = -7{,}2$; $f'(1) = -2{,}8$ hoch 3 + f'

$$\begin{array}{ccc} a & b & c \\ \end{array}$$

$$\left(\begin{array}{ccc|c} 24 & 12 & 6 & -6 \\ -30 & -6 & 0 & 8 \\ -12 & 0 & 0 & 22 \end{array}\right)$$

$-12a = 22 \;|:-12$

$a = \dfrac{22}{-12} = -\dfrac{11}{6}$

$-30 \cdot \dfrac{-11}{6} - 6b = 8$

$55 - 6b = 8 \;|-55$

$-6b = -47 \;|:(-6)$

$b = \dfrac{+47}{6}$

$24 \cdot \left(-\dfrac{11}{6}\right) + 12 \cdot \dfrac{47}{6} + 6c = -6$

$-44 + 94 + 6c = -6$

$50 + 6c = -6 \;|-50$

$6c = -56 \;|:6$

$c = -\dfrac{28}{3}$

---

weitere Beispiele

a) $f(-4)=0{,}6$; $f(2)=0{,}6$; $f(1)=-0{,}4$ hoch 2

b) $f(0)=-1$; $f(2)=15$; $f(-1)=-15$; $f(3)=41$ hoch 3

c) $f(3)=11$; $f(0)=2$; $f'(1)=1$ hoch 2 + f'

d) $f(0)=-3$; $f(2)=-7{,}8$; $f(3)=-7{,}2$; $f'(1)=-2{,}8$ hoch 3+f'

a) $16a - 4b + c = 0{,}6$
$4a + 2b + c = 0{,}6$
$a + b + c = -0{,}4$

$$\left(\begin{array}{ccc|c} 16 & -4 & 1 & 0{,}6 \\ 4 & 2 & 1 & 0{,}6 \\ 1 & 1 & 1 & -0{,}4 \end{array}\right)$$

b) $d=-1$
$8a + 4b + 2c = 16$
$-a + b - c = -14$
$27a + 9b + 3c = 42$

$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & 16 \\ -1 & 1 & -1 & -14 \\ 27 & 9 & +3 & 42 \end{array}\right)$$

c) $9a + 3b + c = 11$
$c = 2$
$2a + b = 1$

$$\left(\begin{array}{cc|c} 9 & 3 & 9 \\ 2 & 1 & 1 \end{array}\right)$$

d) $d=-3$
$8a + 4b + 2c = -4{,}8$
$27a + 9b + 3c = -4{,}2$
$3a + 2b + c = -2{,}8$

$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & -4{,}8 \\ 27 & 9 & 3 & -4{,}2 \\ 3 & 2 & 1 & -2{,}8 \end{array}\right)$$